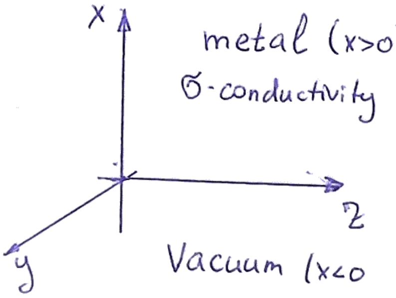
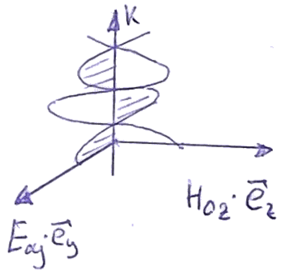
Normal skin-effect

Let us consider penetration of the oscillating electro-magnetic field, i.e. electromagnetic wave, into the metal.



Maxwell's equations:

$$\begin{cases} \operatorname{rot} \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & \text{(in non-magnetic metal, } \vec{B} = \mu_0 \vec{H}) \\ \operatorname{rot} \vec{H} = \vec{j}_f + \dfrac{\partial \vec{D}}{\partial t} & \text{(in metal, } \frac{\partial \vec{D}}{\partial t} = 0 \text{ and } \vec{j}_f = \sigma \vec{E}) \\ \operatorname{div} \vec{D} = \rho_f \\ \operatorname{div} \vec{B} = 0 \end{cases}$$



reminder: $\operatorname{rot} \vec{A} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \vec{e}_x \cdot \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \vec{e}_y \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \vec{e}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right).$

We are looking for a solution: $\vec{E} = E_{0y} \cdot \vec{e}_y \cdot e^{i(Kx - \omega t)}$

$\vec{H} = H_{0z} \cdot \vec{e}_z \cdot e^{i(Kx - \omega t)}$

$$\operatorname{rot} \vec{E} = \vec{e}_z \cdot \frac{\partial E_y}{\partial x} = \vec{e}_z \cdot E_{0y} \cdot (iK) \cdot e^{i(Kx - \omega t)} = -\mu_0 \vec{e}_z \cdot H_{0z} \cdot e^{i(Kx - \omega t)} \cdot (-i\omega) \Rightarrow \underline{E_{0y} \cdot K = \mu_0 \cdot \omega \cdot H_{0z}} \quad (1)$$

$$\operatorname{rot} \vec{H} = -\vec{e}_y \cdot \frac{\partial H_z}{\partial x} = -\vec{e}_y \cdot H_{0z} (iK) e^{i(Kx - \omega t)} = \sigma \cdot E_{0y} \cdot \vec{e}_y e^{i(Kx - \omega t)} \Rightarrow \underline{E_{0y} \cdot \sigma = -iK \cdot H_{0z}} \quad (2)$$

$$(1) : (2) \quad \frac{K}{\sigma} = \frac{\mu_0 \omega}{-iK} \Rightarrow K^2 = i \cdot \mu_0 \sigma \omega \Rightarrow K = \sqrt{\frac{\mu_0 \sigma \omega}{2}} (1 + i) = K_1 + i K_2$$

Substituting this in $\vec{E} \propto e^{i(Kx - \omega t)}$:

$$\vec{E} = E_{0y} \vec{e}_y \cdot e^{i(K_1 x - \omega t)} \cdot e^{-x/\delta}$$

$$k_1 = \sqrt{\frac{\mu_0 \sigma \omega}{2}}, \quad \delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}}$$ ← skin depth, over wich the e-m wave penetrates inside the metal.

for Cu: $\nu = 50\ \text{Hz}$: $\delta = 9.34\ \text{mm}$

$\nu = 10^{10}\ \text{Hz}$: $\delta = 2.1 \cdot 10^{-5}\ \text{mm}$.
(microwave)

if $\delta \lesssim l$ (l - mean free path of an electron), one has to consider anomalous skin effect.